B.Sc- part-II paper-III. Lectivered By Dr. H.K. Yadav.
Deptt. of Mathematics. Marwari college, DBG. Date-12-05-2020
Diff. calculus (Partial differentiation Euler theorem)

Q.1. Verify Euler's Theorem when $u = \frac{x(x^3-y^3)}{x^2+y^2}$

Soln: It is given that $u = \frac{x(x^3-y^3)}{x^3+y^2}$

Taking logarithm on both sides, we get

$$\log u = \log x + \log(x^3-y^3) - \log(x^2+y^2)$$

Diff. w.r.t. x, we get

$$\frac{1}{u}\cdot\frac{\partial u}{\partial x} = \frac{1}{x} + \frac{1}{x^3-y^3}\times 3x^2 - \frac{1}{x^3+y^3}\times 3x^2$$

$$\therefore \frac{1}{u}x\frac{\partial u}{\partial x} = 1 + \frac{3x^3}{x^3-y^3} - \frac{3x^3}{x^3+y^3} \quad \text{———} \boxed{\text{I}}$$

Also, Diff. w.r.t. y, we get

$$\frac{1}{u}\frac{\partial u}{\partial y} = \frac{-3y^2}{x^3-y^3} - \frac{3y^2}{x^3+y^3}$$

$$\therefore \frac{1}{u}y\frac{\partial u}{\partial y} = \frac{-3y^3}{x^3-y^3} - \frac{3y^3}{x^3+y^3} \quad \text{———} \text{II}$$

Adding I and II, we get

$$\frac{1}{u}\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right) = 1 + \frac{3\cancel{(x^3-y^3)}}{\cancel{x^3-y^3}} - \frac{3\cancel{(x^3+y^3)}}{\cancel{x^3+y^3}} = 1+\cancel{3}-\cancel{3} = 1$$

$$\therefore x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$$

When the given function $u = \frac{x^{\cancel{4}}\left(1-\frac{y^3}{x^3}\right)}{\cancel{x^3}\left(1+\frac{y^3}{x^3}\right)} = x\frac{\left(1-\frac{y^3}{x^3}\right)}{\left(1+\frac{y^3}{x^3}\right)} = x\phi\left(\frac{y}{x}\right)$

is a homogeneous function of degree one, therefore u satisfies Euler's Theorem.

Q.2. Verify Euler's Theorem when $u = x^3 \log\frac{y}{x}$.

Soln We have $u = x^3\log\frac{y}{x}$

$$\Rightarrow u = x^3(\log y - \log x)$$

Diff w.r.t. x, we get

$$\frac{\partial u}{\partial x} = 3x^2(\log y - \log x) - x^3\cdot\frac{1}{x}$$

12

$\therefore x\dfrac{\partial u}{\partial x} = 3x^3(\log y - \log x) - x^3$ —— (I)

Also, ~~$\dfrac{\partial u}{\partial y} = x^3$~~

Again, diff. w.r.t. y, we get

$\dfrac{\partial u}{\partial y} = x^3.\dfrac{1}{y} \Rightarrow y\dfrac{\partial u}{\partial y} = x^3$ —— II

Adding I and II, we get

$\therefore x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 3x^3(\log y - \log x) = 3u$

Now, $u = x^3 \log\dfrac{y}{x} = x^3\phi$

Here, u is a homogeneous function of degree 3.

Hence, the given function satisfies Euler's Theorem.

Q3. Verify Euler's Theorem for the expansion

$x^n \sin\dfrac{y}{x}$.

Soln: Let $u = x^n \sin\dfrac{y}{x}$, we get

Diff. w.r.t. x, we get

$\dfrac{\partial u}{\partial x} = nx^{n-1}\sin\dfrac{y}{x} - x^n\cos\dfrac{y}{x} \times \dfrac{y}{x^2}$

$\therefore x\dfrac{\partial u}{\partial x} = nx^n\sin\dfrac{y}{x} - x^{n-1}y\cos\dfrac{y}{x}$ —— (I)

Again, diff. w.r.t. y, we get

$\dfrac{\partial u}{\partial y} = x^n\cos\dfrac{y}{x}\cdot\dfrac{1}{x}$

$\therefore y\dfrac{\partial u}{\partial y} = y.x^{n-1}\cos\dfrac{y}{x}$ —— (II)

Adding I and II, we get

$x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = nx^n\sin\dfrac{y}{x} = nu$

The given function $x^n\sin\dfrac{y}{x}$ is a homogeneous function of degree n.

Hence, the given function satisfies Euler's Theorem.